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Investigation on the Torrance-Sparrow Specular BRDF Model

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ABSTRACT

To describe the directional reflectance characteristics of a specularly reflecting rough surface, K.E. Torrance and E. Sparrow developed a BRDF model based on geometrical optics. In this study their model is confirmed by a ray-tracing-like simulation, using the same assumptions as in the analytical derivation. We also present a simple empirical function that captures the basic features of the analytical model quite well and has been used successfully in BRDF inversion problems including surfaces like roof cover materials and grass canopy.

INTRODUCTION

The 'Bidirectional Reflectance Distribution Function' BRDF as defined by [1] is commonly used to describe the bidirectional reflectance of diffusely scattering surfaces:

$$f_r(\theta_i, \phi_i, \theta_r, \phi_r) \equiv \frac{dL_r(\theta_i, \phi_i, \theta_r, \phi_r)}{dE_i(\theta_i, \phi_i)} \qquad (1)$$

(L = radiance, E = irradiance, index r = reflected, i = incident, θ = zenith angle, ϕ = azimuth angle.) In the case of a perfect specular scattering surface, the BRDF becomes infinite in the specular direction and thus difficult to use. However, in remote sensing perfectly scattering surfaces are very rare. Usually the scattering surfaces are not perfectly smooth, so the reflected radiance is not scattered into one direction only. This makes the reflectance characteristics of the specular surface describable by the BRDF without having to deal with infinities. Torrance and Sparrow [2] developed a model (called TS model in the following) to describe the specular peak of rough surfaces. This model is still used in modern applications [3]. In this paper we confirm the model by numerical ray-tracing-like simulations and present a simple empirical function to describe the BRDF of a specular peak. This function can easily be used in BRDF inversion problems.

BRDF OF A 'PERFECT' SPECULAR PEAK

We define the angle ψ as the angle relative to the specular direction (see fig. 1):

$$\psi \equiv \cos^{-1}(-\sin\theta_i \sin\theta_r \cos\phi + \cos\theta_i \cos\theta_r) \qquad (2)$$
$$\psi \,\epsilon\, [0°, 180°], \qquad \phi \equiv |\phi_i - \phi_r|$$

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In the specular direction $(\theta_i = \theta_r, \phi = 180°)\, \psi = 0°$, at e.g. $(\theta_i = 40°, \theta_r = 50°, \phi = 180°)$ one obtains $\psi = 10°$. The mathematically correct form of a 'perfect' specular peak is a delta function : $f_r \propto \delta(\psi)$. In this section a continuous function (without infinite values as in $\delta(\psi)$) for the specular peak will be developed phenomenologically.

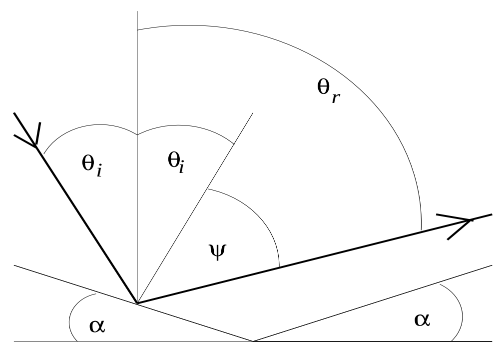

Fig.1: Definition of the angles in the principal plane. The thick line indicates the light beam with an incident zenith angle θ_i and a view zenith angle θ_r. The relative angle to the specular direction $(\theta_r = \theta_i)$ is ψ. The surface patches of the V-cavity have an inclination α.

A change in θ_r will change the area seen by the sensor by a factor of $\frac{1}{\cos\theta_r}$ (or $\frac{1}{\cos\theta_i}$, as $\theta_i = \theta_r$ in the specular direction). As for any surface, the irradiance becomes 0 for zenith angles towards $\theta_i = 90°$, the well known cosine dependence $(E_i = \pi L_i \cos\theta_i)$. As a perfect specular surface reflects all the irradiance, the reflected radiance L_r also has to be proportional to $L_i \cos\theta_i$.

The delta function will be approximated here by a Gaussian exponential function $\frac{1}{\sqrt{2\pi}\sigma} \cdot e^{-\frac{\psi^2}{2\sigma^2}}$. To obtain a very sharp peak, σ has to be chosen very small, the normalization constant becomes very large. The resulting function has the form

$$f_r(\theta_i, \theta_r, \phi) \propto \frac{L_i \cos\theta_i}{L_i \cos\theta_i} \cdot \frac{1}{\cos\theta_r} \cdot \frac{e^{-\frac{\psi^2}{2\sigma^2}}}{\sqrt{2\pi}\sigma} \propto \frac{e^{-\frac{\psi^2}{2\sigma^2}}}{\sigma\cos\theta_r} \qquad (3)$$

This function does not obey Helmholtz's theorem of reciprocity (changing the angles of incidence and reflection changes the BRDF). We substituted the term $\frac{1}{\cos\theta_r}$ with $\frac{1}{\sqrt{\cos\theta_i \cos\theta_r}}$. In the case of a very sharp peak, this yields the same BRDF as before, because $\theta_i \approx \theta_r$ for all BRDF values not equal to zero.

TORRANCE-SPARROW MODEL

For rough surfaces, the specular peak is not centered around the direction $\theta_r = \theta_i$, but shifted towards large zenith angles. [2] proposed a BRDF model based on geometrical optics explaining this shift. They assumed that the surface consists of specularly reflecting V-cavities of infinite length (this inconsistency has been critized by [4] recently, but the model proposed by [4] yields similar results, so the approach of [2] seems to be a good approximation). In the following, we will not treat the diffuse component of the TS model but focus on the specular part only.

The distribution of the *inclinations* of the surface patches is assumed to be Gaussian by TS. This produces a specular peak of Gaussian shape. For large zenith angles, masking and shadowing will influence the contribution of the surface patches to the specular peak. This is accounted for by a so-called 'Geometric-Attenuation-Function' G. The Fresnel reflectance F is a function of the complex index of refraction $\hat{n}$ and the angle of incidence of the specularly reflecting surface patch. This yields the following relation for the BRDF of the specular peak:

$$f_r \propto \frac{F(\theta_i, \theta_r, \phi, \hat{n}) \cdot G(\theta_i, \theta_r, \phi) \cdot e^{-c^2\alpha^2}}{\cos\theta_i \cos\theta_r} \qquad (4)$$

where $\alpha = \alpha(\theta_i, \theta_r, \phi)$ is the angle of the specular reflecting surface patch normal to nadir and g is a constant determining the intensity of the specular reflection.

The function is plotted for several angles in fig. 2. It can be seen that even for moderately rough surfaces ($c = 0.05$) the specular peak is not limited to the ideal specular direction ($\theta_i = \theta_r, \phi = 180°$). E.g. at nadir viewing ($\theta_r = 0°$) and an incident zenith angle of $\theta_i = 45°$ the intensity of the specular peak is still about 20 % of its value at ideal specular direction.

RAY-TRACING VALIDATION

We have confirmed the above model by a ray-tracing-like simulation as used in [5]. The simulation predicts numerically the BRDF of a surface of one-dimensional roughness (in this case parallel V-cavities). It accounts exactly for masking and shadowing for single scattering. The BRDF of the surface patches is needed as input for our simulation program. We used a specular peak of the form described in (3). For $c = 0.01\,\text{deg}^{-1}$ and $c = 0.05\,\text{deg}^{-1}$ we set $\sigma = 0.07°$, for $c = 0.15\,\text{deg}^{-1}$ (a very smooth surface) we set $\sigma = 0.03°$ (corresponding to an even sharper peak). The Fresnel reflectance F was taken as constant.

[2] arranged the surface patches in V-cavities of infinite length. We summed up the reflection contributions predicted by the simulation program for every possible inclination (zenith angle α) and orientation (azimuth angle Φ) of the V-cavities weighted by the inclination distribution of the surface patches according to [2]. This way we simulated exactly the same surface as in the TS model.

The results are shown in fig. 2. The agreement between the TS model and our simulation is very good. This is not surprising, as the same assumptions were made for both models. However, the approaches to the problem are so different (analytical calculation in the TS model versus ray-tracing-like simulation) that the excellent agreement of the two curves is an independent confirmation of the correct BRDF of specular reflecting V-cavities of infinite length derived by [2].

The strongest deviations of the simulated values from the TS model occur for $c = 0.15$ (at $\theta_i = 75°$). This corresponds to a quite smooth surface, in our simulation the width σ of the peak becomes a critical parameter. The larger the parameter c is chosen, the finer the angular grid for the surface patch inclination $(\alpha, \Phi))$ has to be chosen, which increases computing time. So these deviations are due to numerical problems of our simulation and do not indicate an approximation or even fault in the TS model.

EMPIRICAL MODEL

Using the TS model is not as straightforward as it might look, for some combinations of angles the formulas given in [2] are not sufficient. For some applications a simpler model is desirable, e.g. in remote sensing usually the Fresnel reflectance F is not known a priori. We will present an empirical model that is computationally ≈ 6 times faster, easier to implement and still captures the basic features. This empirical model performed well in several BRDF inversion problems including surfaces like roof cover materials and grass canopy, see [5], [6], [7]. Using ψ as defined in (2), the empirical model can be written as:

$$f_r = a \cdot e^{b\cdot(\theta_i\theta_r)^2} \cdot e^{-c^2(\frac{\psi}{2})^2} \qquad (5)$$

where a describes the intensity of the peak, b the shift towards large zenith angles and c the width of the specular peak. In the principal plane $2\alpha = \psi$, so the parameter c in (5) corresponds to the parameter c in the TS model (4). For a constant Fresnel reflectance, we found that the best agreement between the TS model and the empirical function is obtained when choosing parameter $b \approx 0.9 \cdot 10^{-7}\,\text{deg}^{-4}$ if angles are given in degrees ($b = 1.0\,\text{rad}^{-4}$ if angles are given in radians). If the Fresnel reflectance depends on the zenith angle (which can hardly be excluded for remote sensing applications), b is another parameter that has to be inverted (additional to a and c). A fitted value $b > 1.0\,\text{rad}^{-4}$ will indicate a Fresnel reflectance rising with high zenith angles. A value b much smaller than $1.0\,\text{rad}^{-4}$ indicates a problem with the inversion, because usually the Fresnel reflectance does not drop significantly for high zenith angles. In practical applications, another possible reason for a deviation of the inverted parameter b from $1.0\,\mathrm{rad}^{-4}$ is a non-Gaussian inclination distribution of the surface patches (e.g. the leaf surface distribution of grass cannot be assumed Gaussian, because grass is a rectophile canopy). Further studies on this point are needed.

As can be seen in fig. 2, the empirical model fits quite well to the TS model, especially for $\theta_i = 45°$ and $\theta_i = 75°$ in the principal plane. The most important deviation from the TS model in the principal plane occurs for $c = 0.01$ (very rough surface) and $\theta_i = 10°$: the TS model rises for large θ_r, but the empirical model is almost constant. At $\phi = 150°$, TS model and empirical model do not agree well for very smooth surfaces (c large), but at $c = 0.15$ the intensity has dropped to less than 10 % of its maximum value, so the absolute deviation is quite small. In general, for angles not in the principal plane the TS model predicts values lower than the empirical model.

CONCLUSIONS AND OUTLOOK

The analytical TS model which describes the BRDF of a specular reflecting rough surface is confirmed by a ray-tracing-like simulation. We present a simple empirical BRDF that captures the basic features of the TS model quite well and can conveniently be used in BRDF inversion problems.

We are planning to investigate on the assumption of a Gaussian inclination distribution for the surface patches. First topography measurements on a roofing tile showed significant deviations from a Gaussian distribution.

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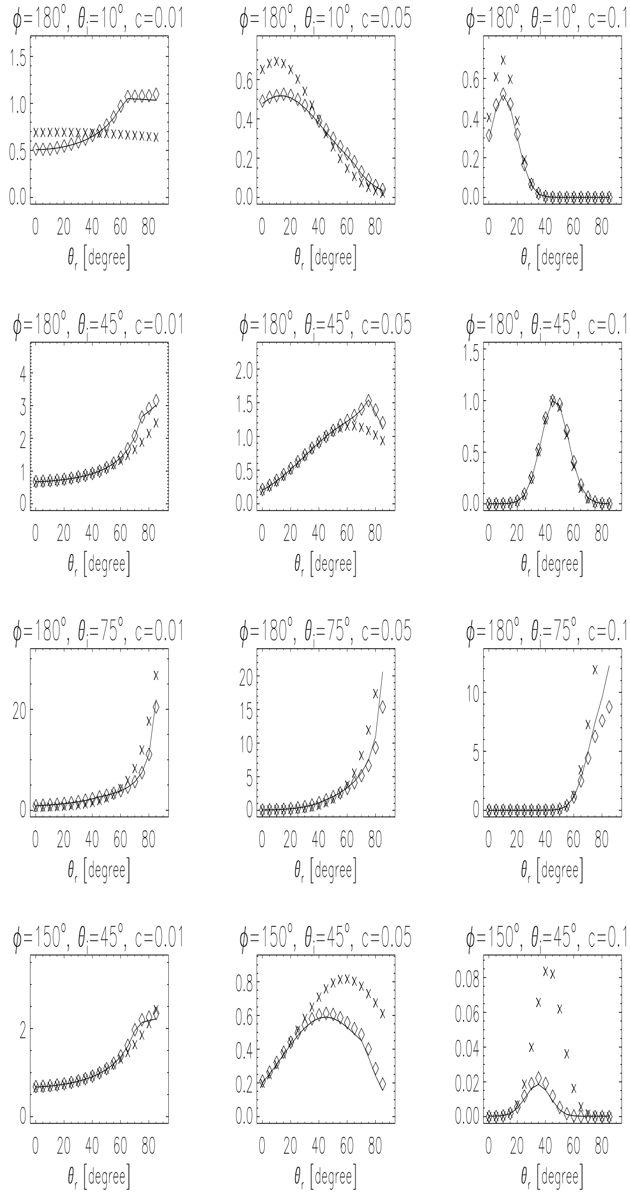

Fig.2: Specular peak BRDF as a function of view zenith angle θ_r. **All BRDF values have been normalized** *to the value at $f_r(\theta_i = \theta_r = 45°, \phi = 180°, c)$, therefore the ordinate is dimensionless. The solid line is the TS model (with index of refraction $\hat{n} = 1$), the rhombs indicate the values obtained by the ray-tracing-like simulation, the crosses are the empirical model (5). The first column on the left has $c = 0.01\,deg^{-1}$, corresponding to a very rough surface, for the second column $c = 0.05\,deg^{-1}$, for the third column on the right $c = 0.15\,degree^{-1}$ (very smooth surface). The first 3 lines show the principal plane ($\phi = 180°$) for $\theta_i = 10°, 45°, 75°$ resp., for the lowest line $\phi = 150°$ and $\theta_i = 45°$. The TS model and the simulated values agree very well, the rhombs almost always lie on the solid line. The empirical model shows some strong deviations from the TS model.*