Modeling Covariance Matrices in Multitemporal Temperature Feature Spaces

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Abstract – Multitemporal thermal imagery in conjunction with a diurnal temperature model provides a well-known means to derive physical properties of the earth's surface. Classification applications based on such data sets can be used for clustering image pixels with similar heating behaviour. For that purpose, the covariance matrix which describes the mutual dependence of the used features (i.e. temperature values measured at different times of a day) provides an important source of information.

In this study we investigate the effect of Gaussian distributed surface properties (thermal inertia, humidity and albedo) on covariance matrices in multitemporal temperature feature spaces with the aid of a diurnal temperature model [1], [2]. The results are compared with experimental findings.

INTRODUCTION

In the last three decades multitemporal thermal data has been used widely to determine surface properties, i.e. mainly thermal inertia and humidity [1],[3]. From a statistical point of view a multitemporal data set of temperature values can be regarded as a multidimensional feature space and the determination of surface properties as a classification process. For classification purposes the mutual dependencies of the features, expressed by covariance matrices, play a major role. Other than in the common case of a feature space formed by reflectance data the covariance matrices are to some extent predictable through the use of diurnal temperature models.

In this contribution we investigate the effect of normal-distributed spatial variations in surface properties on covariance matrices in multitemporal temperature feature spaces. We therefore use a diurnal temperature model [1],[2] with simplified expressions for the incoming and outgoing heatfluxes which represent the influence of atmospheric conditions and surface properties on the thermal behaviour. The results are compared with experimental findings based on a remotely sensed multitemporal thermal image data set recorded by the airborne multispectral scanner DAEDALUS AADS 1268 over an urban area. Image data sets were taken at four times of one summer day (4:30, 8:30, 11:30, 12:30 MET). Finally we comment on the usefulness of modeled covariance matrices for the improvement of unsupervised classification.

THEORY

In a statistical approach we regard a multitemporal set of measured or modeled temperature values of one surface type as a set X_a of n_a vectors x belonging to a class a. The elements of x are the temperature values at different times. Assuming a multivariate Gaussian normal distribution the *covariance matrix* Σ_a can be estimated as

$$\Sigma_a = \frac{1}{n_a} \sum_{x \in X_a} (x - m_a)(x - m_a)^{\mathrm{T}} \qquad (1)$$

with $m_a = \frac{1}{n_a} \sum_{x \in X_a} x$. Each of the n_a^2 elements (unit: $[Kelvin^2]$) of the covariance matrix describes the mutual dependence of two temperature values at different times.

With the aid of diurnal temperature models the effect of surface parameter fluctuations on the covariance matrix can be understood and the matrices can be predicted to some extent. The selection of an appropriate model for these investigations was based on the following considerations:

- the model must be flexible in handling boundary conditions and should be able to take into account the commonly assumed case of a uniform semi-infinite halfspace as well as an additional surface layer cover;
- the model must be represented by a fast and reliably converging algorithm since for the calculation of variances a large number of model runs is needed.

Analytical models based on Fourier analysis lack the first property [4]. Numerical models based on finite differences methods show a convergence behaviour which is too slow for our interests [5].

A modeling technique fulfilling both demands is the use of the Laplace transform as described in [1] and [2]. Application of the Laplace transform to the one-dimensional heat conduction equation yields the following equation:

$$F_{\mathrm{n}} = \frac{P}{\pi T} \sum_{s=1}^{m} V_{\mathrm{s}} \phi_{\mathrm{n-s+1}} \qquad (2)$$

where F_n is the flux into the ground in the n^{th} (time) interval, P the thermal inertia, $T = 24\mathrm{h}$, V_s the surface temperature in the s^{th} interval and $\phi_{\mathrm{n-s+1}}$ a set of coefficients determined solely by the number of incremental (time) steps m in the case of a semi-infinite solid. Assuming a layer of different thermal properties (denoted by the subscript 'l') over a semi-infinite halfspace they additionally depend on the layer's thickness d_{l}, the layer's thermal diffusity k_{l} and the ratio of the two thermal inertia values P/P_{l}. The complete expressions for ϕ are given in [1] for the case of a semi-infinite solid and in [2] for the case of a layer over a semi-infinite solid.

Applying the commonly used boundary condition (balance of incoming and outgoing heatfluxes at the surface) to (2) yields a nonlinear equation:

$$F_\mathrm{n} = F_n^\mathrm{sun} - F_n^\mathrm{surf} + F_n^\mathrm{sky} - F_n^\mathrm{sens} - F_n^\mathrm{lat} \qquad (3)$$

with $F_n^\mathrm{sun} = S_0 V(1-al)g(t_\mathrm{n})$ incoming solar radiation (al surface albedo, $S_0 = 1375\ \mathrm{W/m^2}$ solar constant, V atmospheric transmittance, $g(t) = \sin\delta\sin\phi + \cos\delta\cos\phi\cos\omega t$ with the condition $g(t) \geq 0$, δ solar declination, ϕ latitude, $\omega = 2\pi/24$h and t_n mean time of the n^th interval, measured from local noon), $F_n^\mathrm{surf} = \epsilon\sigma T_\mathrm{s}^4$ the emitted longwave radiation (ϵ surface emissivity, σ Boltzmann constant, T_s surface temperature) and F_n^sky the incoming sky radiation. The expressions for the sensible heatflux F_n^sens and the latent heatflux F_n^lat are used in their most basic forms:

$$F_n^\mathrm{sens} = \frac{\rho_\mathrm{a} c_\mathrm{p}(T_\mathrm{s} - T_\mathrm{a})}{r} \qquad (4)$$

$$F_n^\mathrm{lat} = \frac{\rho_\mathrm{a} L(mq_\mathrm{sat} - q_\mathrm{a})}{r} \qquad (5)$$

with ρ_a density of the air, $c_\mathrm{p} = 1010$ J/kgK heat capacity of air, $L = 2.5 * 10^6$ J/kg latent heat of evaporation, T_a air temperature, q_sat saturated specific humidity at the surface, q_a specific humidity of the air, $m \in [0,1]$ surface relative humidity and $r = (\ln(z_1/z_0))^2/\kappa^2/u$ aerodynamic resistance with z_0 surface roughness, z_1 height of wind speed observation, $\kappa = 0.42$ the Karman constant and u windspeed.

Having specified all needed parameters, (2) and (3) can be solved for the surface temperature T_s for each time step using conventional iteration techniques. On a SparcUltraI with 147 MHz and 160 MB RAM the calculation of temperatures for a diurnal cycle in steps of 5 minutes takes between 10 and 15 seconds, which is tenable for our investigations.

GENERALIZATION OF MODEL PARAMETERS

Our goal is to understand and predict the covariance matrices for some typical surface types in urban environments by using the model described above. The usefulness of modeled covariance matrices, e.g. for unsupervised classification applications, is reduced dramatically if we have to rely on tedious and costly measurements of surface and atmospheric parameters. So we demand a high degree of independence from actual surface and atmospheric conditions. To achieve this we have to define the parameters in (3) – (5) needed to run the model in a very general sense for all types of analyzed surfaces. The accuracy loss regarding *absolute* temperature values due to imprecise values of surface properties is tenable since we are only interested in temperature *variances*.

The following parameters were calculated according to the *midlatitude summer atmosphere* given in [6]:

$V = 0.75$, $q_a = 0.0118$, $q_\mathrm{sat} = 0.0155$.

These parameters are considered to be typical for our testsite:

$u_0 = 3$ m/s, $z_1 = 2$ m, $\rho_a = 1.16\ \mathrm{kg/m^3}$, $\epsilon = 0.95$.

As air temperature and humidity depend strongly on the surface temperature and there is no simple analytical solution to describe diurnal variations of these two parameters we make the following assumption:

$$F_n^\mathrm{sens} + F_n^\mathrm{lat} = \frac{\rho_\mathrm{a}}{r}(L(mq_\mathrm{sat} - q_\mathrm{a}) + c_\mathrm{p} * 10°\mathrm{C}) * g(t_\mathrm{n})$$

with $g(t)$ as defined above. Of course this assumption is very rough, but since we are only interested in temperature *variances* it is tenable. The atmospheric counter radiance F^sky is modeled as $F^\mathrm{sky} = \sigma T_\mathrm{sky}^4$ with $T_\mathrm{sky} = 271°\mathrm{C} + 10°\mathrm{C} * \cos(\omega(t_\mathrm{n} - 2\mathrm{h}))$ following the approach of [7] with an adaption of the first addend to the T_sky-value defined in the midlatitude summer atmosphere.

The parameters which must still be specified to adapt the modeled covariance matrix to a special surface type are:

z_0, P, m, al

and in the case of a surface layer additionally:

d_l, k_l, P/P_l.

METHOD

We assume that the variablity of temperature behaviour whithin one surface type stems from the Gaussian fluctuation of three parameters: **thermal inertia** P, **moisture content** m and **albedo** al. It is well-known that P largely affects the amplitude of surface temperatures while m and al bias the daily mean temperature (see e.g. [3]). Thus, to examine the effect on the covariance matrix of a surface type the assumed mean values of these three parameters have to be defined and a statistically sufficient number of calculations of diurnal temperature curves have to be made with all parameters kept constant except P, m and al. For these parameters we choose Gaussian distributions with standard deviations

σ_P, σ_m, σ_{al}

that have to be specified according to the characteristics of the examined surface type (e.g. albedo fluctuations of grasslands are in general much greater than those of streets). We assume 100 model runs to be sufficient to estimate the variances.

RESULTS

To test our model we use a multitemporal thermal data set recorded at the 26 August, 1997, near and at the city of Nuremberg, Germany (solar declination $\delta = 49.5°$, latitude $\phi = 10.3°$). The recording system was a DAEDALUS AADS 1268 line scanner on board a Dornier Do 228 aircraft. Atmospheric corrections were performed by using the SENSAT/MODTRAN computer code [8] relying on standard atmospheric parameters supplemented by ground measurements of meteorological data and radiosonde data recorded in Amberg (located 60 km in the east of Nuremberg) [9]. The recording times were 4:30, 8:30, 11:30 and 12:30 MET (we omitted the 11:30-measurement in the following investigations since it is very similar to that of 12:30). From this data set we defined several regions showing streets and unsealed surfaces and calculated the averaged covariance matrices with (1). The feature vectors x are three-dimensional vectors whose elements are the surface temperature values measured by the sensor at 4:36, 8:22 and 12:22 MET. The resulting matrices are 3×3-matrices.

$$\Sigma^{meas.}_{unsealed} = \begin{pmatrix} 0.06 & 0.03 & 0.04 \\ 0.03 & 0.84 & 0.81 \\ 0.04 & 0.81 & 2.71 \end{pmatrix}$$
$$\Sigma^{meas.}_{street} = \begin{pmatrix} 0.07 & 0.00 & -0.01 \\ 0.00 & 0.40 & 0.08 \\ -0.01 & 0.08 & 0.29 \end{pmatrix}$$

For the corresponding modeling we assume the values of the 'free' parameters as follows:

- unsealed (halfspace, moist and smooth surface, no fluctuation in thermal inertia, small fluctuations in humidity, large fluctuations in albedo): $P = 1000\,\mathrm{Jm^{-2}s^{-1/2}K^{-1}}, m = 0.9, al = 0.15, z_0 = 0.005$ m, $\sigma_P = 0.0, \sigma_m = 0.03, \sigma_{al} = 0.10 * al$,
- street (halfspace with layer at double thermal inertia, dry and smooth surface, small fluctuations in thermal inertia and albedo): $P = 1000\ \mathrm{Jm^{-2}s^{-1/2}K^{-1}}$, $m = 0.9$, $al = 0.15$, $z_0 = 0.005$ m, $k_l = 9 * 10^{-7}$ m^2/s, $d_l = 20$ cm, $P/P_l = 0.45$, $\sigma_P = 0.03 * P$, $\sigma_m = 0.0$, $\sigma_{al} = 0.03 * al$.

The resulting covariance matrices are:

$$\Sigma^{mod.}_{unsealed} = \begin{pmatrix} 0.27 & 0.36 & 0.73 \\ 0.36 & 0.53 & 1.01 \\ 0.73 & 1.01 & 1.97 \end{pmatrix}$$
$$\Sigma^{mod.}_{street} = \begin{pmatrix} 0.09 & -0.02 & -0.08 \\ -0.02 & 0.05 & 0.10 \\ -0.08 & 0.10 & 0.23 \end{pmatrix}$$

The similarity of some of the modeled and measured matrix elements encourages our modeling approach, but further refinement is needed in the specification of the parameters to obtain more accurate results.

COMMENT ON THE APPLICATION TO CLASSIFICATION ALGORITHMS

As the amount of available remotely sensed data increases steadily, unsupervised classification techniques become more and more important. An improvement of classification accuracies can be obtained by taking into account prior knowledge. In the special case of a multitemporal temperature feature space covariance matrices of different surface types can be modeled based on a diurnal temperature model with generalized parameters and very few assumptions on actual surface parameters. Whether this modeling can yield matrices precise enough to abandon the often tedious training phases in supervised classification applications is subject of our actual work.

SUMMARY AND CONCLUSION

We investigated the effect of Gaussian distributed surface properties on covariance matrices in multitemporal temperature feature spaces. The results show the possibility to use this modeling approach for unsupervised classification applications on multitemporal thermal data.

ACKNOWLEDGMENT

This work was supported by the Volkswagen-Stiftung. The image flights were conducted in collaboration with the German Aerospace Center (DLR Weßling, München), particularly with the help of Volker Amann and Manfred Schröder. We thank Rudolf Richter (DLR) for discussions and support concerning atmospheric corrections.

REFERENCES

[1] K. Watson, L. C. Rowan, and T. W. Oldfield. Application of Thermal Modeling in the Geologic Interpretation of IR Images. In *Proc. 7th Int. Symp. Remote Sens. Environ.*, pages 2017–2041, 1971.

[2] K. Watson. Periodic Heating of a Layer over a Semi-Infinite Solid. *Journal of Geophysical Research*, 78(26):5904–5910, 1972.

[3] J. C. Price. Estimation of Regional Scale Evapotranspiration Through Analysis of Satellite Thermal Infrared Data. *IEEE Transactions on Geoscience and Remote Sensing*, GE-20(3):286–292, 1982.

[4] J. C. Price. On the Analysis of Thermal Infrared Imagery: The Limited Utility of Apparent Thermal Inertia. *Remote Sensing of Environment*, 18:59–73, 1985.

[5] A. B. Kahle. A Simple Thermal Model of the Earth's Surface for Geological Mapping by Remote Sensing. *Journal of Geophysical Research*, 82(11):1673–1680, 1977.

[6] R. A. McClatchey, W. Fenn, J. E. A. Selby, F. E. Volz, and J. S. Garing. Optical Properties of the Atmosphere. *AFCRL Envirn. Res. Papers*, 411:108pp., 1972.

[7] K. Kondratyev. *Radiation in the Atmosphere*. Academic, New York, 1969.

[8] R. Richter. *Radiometrische Auslegung von Sensoren und quantitative Auswertung von Fernerkundungsdaten im optischen Spektralbereich*. PhD thesis, German Aerospace Research Establishment (DLR), Oberpfaffenhofen, 1992.

[9] B. Prinz. Thermalkartierung städtischer Gebiete anhand von Multispektralbildern , Diplomarbeit, Universität Hamburg, II. Institut für Experimentalphysik, CENSIS-Report 31-98., 1998.

Table 1: Modeled covariance matrices.

$$\Sigma_{P,5\%} = \begin{pmatrix} 0.30 & -0.27 & -0.39 \\ -0.27 & 0.24 & 0.35 \\ -0.39 & 0.35 & 0.52 \end{pmatrix} \quad \Sigma_{P,10\%} = \begin{pmatrix} 1.14 & -1.02 & -1.49 \\ -1.02 & 0.92 & 1.34 \\ -1.49 & 1.34 & 1.95 \end{pmatrix}$$

$$\Sigma_{m,10\%} = \begin{pmatrix} 1.61 & 2.25 & 4.44 \\ 2.25 & 3.14 & 6.20 \\ 4.44 & 6.20 & 12.26 \end{pmatrix} \quad \Sigma_{m,20\%} = \begin{pmatrix} 5.09 & 7.02 & 13.85 \\ 7.02 & 9.68 & 19.09 \\ 13.85 & 19.09 & 37.65 \end{pmatrix}$$

$$\Sigma_{al,10\%} = \begin{pmatrix} 0.11 & 0.27 & 0.39 \\ 0.27 & 0.68 & 0.97 \\ 0.39 & 0.97 & 1.40 \end{pmatrix} \quad \Sigma_{al,20\%} = \begin{pmatrix} 0.38 & 0.96 & 1.38 \\ 0.96 & 2.42 & 3.48 \\ 1.38 & 3.48 & 5.00 \end{pmatrix}$$